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Article (Published Version)

Hudson, Brian (2019) Epistemic quality for equitable access to quality education in school mathematics. *Journal of Curriculum Studies*, 51 (4). pp. 437-456. ISSN 0022-0272

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To cite this article: Brian Hudson (2019) Epistemic quality for equitable access to quality education in school mathematics, Journal of Curriculum Studies, 51:4, 437-456, DOI: [10.1080/00220272.2019.1618917](https://doi.org/10.1080/00220272.2019.1618917)

To link to this article: <https://doi.org/10.1080/00220272.2019.1618917>



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Published online: 20 May 2019.



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# Epistemic quality for equitable access to quality education in school mathematics

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## ABSTRACT

This paper reports on a study that aims to address the challenges of UN Sustainable Development Goal 4 to ensure inclusive and equitable quality education for all. The study focuses on school mathematics in particular. With regard to ensuring equitable access to quality education, it is argued that there is a need to consider the *epistemic quality* of what students come to know, make sense of and be able to do in school mathematics. Accordingly, the aim is to maximize the chances that all pupils will have *epistemic access* to school mathematics of *high epistemic quality*. The study is based on the theoretical framework of Joint Action Theory in Didactics (JATD). Associated research questions focus on the quality of teacher-student(s) joint action and on the epistemic quality of the content. The paper draws on empirical research findings of the *Developing Mathematical Thinking in the Primary Classroom* (DMTPC) project (2010–12) and also on the findings of a parallel study of mathematics teachers' assessment practices in Ghana. One teacher's action research project is used as an exemplar to illustrate how mathematics can become more accessible and inclusive thus leading to an evolution in mathematical thinking and high-quality epistemic access *for all*.

## KEYWORDS

Elementary school curriculum; access to education; educational quality; mathematics education

## 1. Introduction

This paper reports on a study that aims to address the challenges of the United Nations Sustainable Development Goal (SDG) Number 4 (United Nations, 2015) which is to ensure inclusive and equitable quality education for all. The study focuses on school mathematics in particular. It arises from the activities of the WERA International Research Network on Didactics—Learning and Teaching (Hudson, 2017, 2016b; Oduro & Hudson, 2016). The need to address SDG 4 is seen not only at the challenge at the global level but also at the national level for all contemporary societies and educational systems in an age of mass migration and the increasing impact of economic austerity measures in many countries of the world. With regard to ensuring equitable access to quality education, it is argued that there is a need to consider the *epistemic quality* (Hudson, Henderson, & Hudson, 2015) of what students come to know, make sense of and be able to do in school mathematics. Accordingly, the aim is to develop curriculum principles that maximize the chances that all pupils will have *epistemic access* (Morrow, 2008) to school mathematics of *high epistemic quality*. The study is based within in the framework of Joint Action Theory in Didactics (JATD) that foregrounds joint action in didactics by students and teachers (Sensevy, 2011). Within this frame the research questions focus on the quality of teacher-student(s) joint action and also on the epistemic quality of the content. The paper draws on empirical research findings of a project on

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*Developing Mathematical Thinking in the Primary Classroom* (DMTPC) which was funded by the Scottish Government during 2010–12 and also on the findings of a parallel study of mathematics teachers' assessment practices in Ghana. From within an epistemology of 'exemplar' as defined by Kuhn (1962), the role of one teacher's action research project is presented as potentially constituting an 'exemplary' case study of high-quality teacher-student(s) joint action and high epistemic quality of content in school mathematics. This is used to illustrate how mathematics can become more accessible and inclusive thus leading to an evolution in mathematical thinking and high-quality epistemic access for all.

## 2. Background context

### 2.1. Developing mathematical thinking in the primary classroom

The DMTPC project involved working with a group of practising teachers ( $n = 24$ ) who were all participants in a newly developed Masters course that had been designed with the aim of promoting the development of mathematical thinking in the primary classroom. The associated research questions focused on the teachers' confidence, competence, attitudes and beliefs relative to mathematics and their expectations and experiences of the impact on pupil learning arising from this course. Empirical data were drawn from pre- and post-course surveys, interviews and observations of the discussion forums in the online environment. Findings from this study highlight the way the course had a transformational and emancipatory impact on these teachers. Typical examples of feedback from the teachers at the end of the course included comments such as: *I experienced many light bulb moments; the course has had a significant effect on my day to day teaching; I now teach differently because this course has helped me identify what really matters in maths learning; the impact on my own class has been enormous and I have become much more flexible and creative in my teaching.* They also highlight how the 'framing' (Bernstein, 2000) of particular aspects of the curriculum had an oppressive impact on learners in ways that suppressed creativity and limited their exercise of learner autonomy. This was expressed in terms of what is perceived to 'be allowed' or 'not allowed' by both teachers and pupils. It was clear that the weaker framing of Curriculum for Excellence had shifted the locus of control over the selection, sequencing and pacing of what counts as legitimate knowledge towards these teachers. However, the teachers' own experience as learners of mathematics highlighted the impact of the strong framing over the criteria for evaluating or assessing the formal assessment system, especially at the secondary school level. In relation to this aspect, several of these teachers had experienced mathematics as a school subject in very negative ways, involving high levels of 'symbolic violence' (Bourdieu, 1998) and being 'labelled'. Each of the teachers who took part in the interviews described either a symbolic or physical positioning which took place and talked about the consequence of such positioning. For example, Angela who started in the 'top class' for mathematics at secondary school described how she was 'pulled out of the classroom' and 'very quickly shoved into another one' where she was not expected 'to do very much'. Indeed, she described herself as 'definitely one of these people who have labelled themselves as not a maths person'. In particular, the teachers had some very powerful responses to reading the book entitled *The Elephant in the Classroom* (Boaler, 2009, p. 2), in which she writes:

*I have called this book 'The elephant in the classroom' because there is often a very large elephant standing in the corner of maths classrooms. The elephant, or the common idea that is extremely harmful to children, is the belief that success in mathematics is a sign of general intelligence and that some people can do maths and some can't. Even maths teachers (the not so good ones) often think that their job is to sort out those who can do maths from those who can't. This idea is completely wrong and this is why. In many maths classrooms a very narrow subject is taught to children, that is nothing like the maths of the world or the maths that mathematicians use. This narrow subject involves copying methods that teachers demonstrate and reproducing them accurately over and over again. Of course very few people are good at working in such a narrow way, and usually everyone knows which people are good at it and which people are not. But this narrow subject is not mathematics, it is a strange mutated*

*version of the subject that is taught in schools. When the real mathematics is taught instead—the whole subject that involves problem solving, creating ideas and representations, exploring puzzles, discussing methods and many different ways of working, then many more people are successful.*

This process of mutation is seen as a transformation process and in particular as an example of 'didactic transposition' as outlined by Chevallard, [1985] 1991, 2007). The concept of didactic transposition arises from research in the field of French didactics in particular and builds on the seminal work of Brousseau (1997) on 'didactic situations'. The basic principle underpinning this perspective on learning and teaching is that knowledge is not something that is to be taken as simply given and to be explained. Rather, it is the case that 'knowledge is potentially encapsulated in situations, and it is in going through those situations that the pupil, or whoever, can learn' (Chevallard, 2007). This view of learning as 'learning from the situation' is a central principle of French didactics which sees knowledge as built up and transformed or *transposed* in didactic situations. Underpinning this theory is an ecological approach to the social dynamic of knowledge (ibid). The main point in didactic transposition theory is that it considers knowledge as a changing reality, which adapts to its institutional habitat. Accordingly, in relation to the school context, the knowledge in question is not knowledge for acting and solving problems in the social contexts in which it was created and where it is used, but instead is transposed into knowledge to be taught and learned. As highlighted by Schneuwly (2011) the concept of didactic transposition is based on the recognition that there is a 'rupture' between daily life and school, which changes the knowledge profoundly. Recognition of this process of didactic transposition in which knowledge can mutate or be degraded leads directly to the consideration of quality and in particular of epistemic quality which is the focus of [section 4.0](#).

## **2.2. Assessment in mathematics classrooms in Ghana: a study of teachers' practices**

The issue of epistemic quality was highlighted by Oduro (2015) in a parallel study conducted in 2011 of mathematics teachers' assessment practices in Ghana. The study explored key factors affecting assessment practices in mathematics classrooms. In particular, it provides an analysis of teachers' views on assessment in relation to improvement and accountability. It also illuminates the knowledge, attitudes and beliefs of the mathematics teachers and how these relate to practice. Findings from this study illustrate how teachers use both formal and informal assessments in mathematics classrooms although formal assessment dominates practice. They also highlight the ways in which teachers' views about the assessment and their conceptions of the nature of mathematics are related to their classroom practices and how teachers' assessment practices are affected by contextual factors that are related to institutional policies, professional development and classroom conditions. Of particular relevance in relation to the former were the teachers' views about the nature of mathematics and the ways in which these can impact so as to degrade the epistemic quality of school mathematics. The findings arising from this study as a whole were subsequently presented in a paper by Oduro and Hudson (2016) at a symposium organized by the WERA International Research Network on Didactics—Learning and Teaching. The implications for teacher education policy and practice in Ghana were also considered. These relate to the need to improve teacher preparation, the qualification of teachers of mathematics in basic schools, knowledge of the subject mathematics, knowledge of teaching mathematics and assessment literacy. In relation to curriculum planning and development, greater emphasis on 'assessment for learning' was identified as a key priority. Accordingly, there was an associated need for assessment practices to move away from an over-emphasis on traditional summative to a more practical formative assessment. This work brought questions of equity and access to the foreground and led to a deeper consideration of inequality as a core challenge that needs to be at the foreground of our thinking about didactics—learning and teaching.

### 3. Joint action theory in didactics

The theoretical framework adopted in this study arises from the Joint Action in Didactics in Europe (JADE) project which has involved a cross-curricular approach across three countries (UK, France and Germany) as members. The JADE project was formed following discussions on Joint Action Theory in Didactics (JATD) (Sensevy, 2011) at the research workshop on Fachdidaktik within Network 27 Didactics -Teaching and Learning held in 2011 at the European Conference on Education Research (ECER) in Berlin. One of the project's first activities involved participation at the International Colloquium on 'Forms of Education and Emancipation Processes' organized by the Centre for Research in Education and Didactics (CREAD) at the University of Rennes 2 in May 2012. The discussions at this event were organized through a symposium on JATD and the first case study in the project was developed by applying the JATD framework to the analysis of classroom interaction in mathematics as outlined in the paper presented by Hudson (2012). This was based on the DMTPC project that was further discussed at the Symposium on Joint Action Theory in Didactics at ECER 2013 in Istanbul (Hudson, 2013) and subsequently published in *Education and Didactique* (Hudson, 2015).

JATD has developed within the tradition of French didactics, the first theoretical principle of which is elaborated by Sensevy, Schubauer-Leoni, Mercier, Ligozat, and Perrot (2005) and Sensevy (2011) as being that, in order to understand a didactic activity, which denotes an activity where someone teaches and someone learns, you need to understand a *system*, the *didactic system*. This is a system of three subsystems, namely knowledge, the teacher, and the student. The didactic system is seen as indivisible and it is regarded as impossible to grasp the meaning of the teacher's action without understanding the relations between this action, the students' action and the structure of the piece of knowledge at stake. This theoretical principle is consistent with the notion of 'tools for holding complexity' as discussed by Hudson (2002, p. 47/48). The didactic triad is seen as a key tool for the analysis of the complex relations between teacher, student and content in the teaching-studying-learning process, which ideally should be treated as a whole. In turn, JATD is based on an understanding of joint action as a process of reciprocal semiosis (Sensevy, 2011, p. 61) involving the deciphering of verbal and bodily actions (or signs) of others in a particular situation. The argument is well made by (ibid.) that didactic research needs a new paradigm, a paradigmatic shift from an analytic stance to a holistic approach, in which the necessary analytic study is only part of the researcher's work. In this respect, the main purpose of the joint action theory is to grasp the dynamics and the unity of the joint action.

The key concepts drawn from JATD and used in this study are 'game', 'contract' and 'milieu'. In relation to the first of these, the fundamental starting point for the JATD, as described by Loquet (2011), is to consider teaching and learning as a joint action, co-operative and asymmetric, occurring between the teacher and the students based on 'language games' (Wittgenstein, 1997) specific to didactic systems. In particular, three games are identified which might be seen in relation to the three aspects of the didactic triad, i.e. content, teacher and learner. These are the 'epistemic game', the 'didactic game' and the 'learning game', respectively. The epistemic game refers to the game of the professional mathematician, for example, and two aspects of this game are distinguished as follows. On one hand, the *source epistemic game* refers to the human practices that exist outside the didactic situation whilst, on the other, the *actual epistemic game* is based on the analysis of class practices as they occur in-situ. Second, the didactic game relates to the game of the teacher in didactic transactions between teacher and students. These are seen as games of a particular kind, i.e. games in which some specific pieces of knowledge are involved and as collaborative or joint games within joint action (Gruson, Loquet, & Pilet, 2012, p. 65). Third, the learning game is the reciprocal game of the student in relation to the joint game. The second key concept is the didactic contract (Brousseau, 1997) which is used to describe the system of habits, which is largely implicit, between the teacher and the students regarding the knowledge in question. Based on those habits established in the didactic situation, each participant (teacher or student) attributes some expectations to the other(s). The didactic contract provides a common background between the teacher

and the students against which the didactic transactions occur. Finally, the third key concept is 'milieu', which refers to the system of material and symbolic objects in question that corresponds to the new knowledge the students are to acquire. According to this description, the older pieces of knowledge enable the teacher and the students to act jointly whereas the new knowledge involves a kind of resistance to the student's action (Gruson et al., 2012, p. 65). The concept of *epistemic quality* emerged from the process of applying the JATD framework to the analysis of one teacher's action research project as illustrated through consideration of Anna's action research project which follows below.

#### 4. Epistemic quality in school mathematics

The overall aim of the DMTPC project was to develop and implement a postgraduate course of continuing professional learning on the theme of 'Developing Mathematical Thinking in the Primary Classroom' (DMTPC). The course was designed to form the first stage in a Masters level programme in Mathematics Education for Primary Specialists in Mathematics and to be accredited as a free-standing module worth 30 credits leading towards a Postgraduate Certificate in Developing Mathematical Thinking. The Curriculum Development Partnership Group that planned the course included one teacher from each of the participating Local Education Authorities (LEA) and two Local Education Authority advisory staff members. The course design was based on a blended learning model with an online learning environment established in the University Virtual Learning Environment.

The course of study was structured around three key questions, two core texts and an action research project. The key questions were designed to support the participants in critically examining the nature of knowledge and of mathematical knowledge in particular, in focusing on the nature of mathematical thinking and in considering the implications for mathematics teaching. The three key questions were:

- (1) What is mathematics?
- (2) What is mathematical thinking?
- (3) What is good mathematics teaching?

In order to support the course of study, there were two core texts, one by Boaler (2009) and the other by Mason, Burton, and Stacey (2010). In addition, the course assignment was an action research project report.

Consideration of the nature of mathematics as part of the course was framed from the outset, through an opening keynote presentation, by a discussion of the contrasts between *mathematical fallibilism* and *mathematical fundamentalism*. The distinction is made between *mathematical fallibilism* based on a heuristic view of mathematics as a human activity (Lakatos, 1976) and *mathematical fundamentalism*, which describes the transposed 'mutation' version outlined earlier. The former involves an approach that presents mathematics as fallible, refutable and uncertain and which promotes critical thinking, creative reasoning, the generation of multiple solutions and of learning from errors and mistakes. In contrast, the latter is characterized by an approach that presents the subject as infallible, authoritarian, dogmatic, absolutist, irrefutable and certain and also involves rule following of strict procedures and right or wrong answers.

The central role of creative reasoning is considered further in Hudson (2018) by drawing on the work of Lithner (2008) who offers a conceptual framework that compares and contrasts creative and imitative reasoning in mathematics that fits with our distinctions (ibid.) between mathematical fallibilism and mathematical fundamentalism. With regard to imitative reasoning in mathematics, Lithner (ibid.) highlights two aspects; memorized reasoning and algorithmic reasoning. Memorized reasoning is seen to fulfil two conditions. First, the strategy choice is founded on recalling a complete answer and, second, the strategy implementation consists only of writing it down. In relation to algorithmic reasoning and with reference to Brousseau (1997, p. 129), an algorithm is



defined as 'a finite sequence of executable instructions that allows one to find a definite result for a given class of problems.' Similarly, algorithmic reasoning fulfils two conditions. First, the strategy choice is to recall a solution algorithm regarding which the predictive argumentation may be of different kinds but does not necessitate the creation of a new solution. Second, the remaining reasoning parts of the strategy implementation are trivial for the reasoner and only a careless mistake can prevent an answer from being reached. Lithner (ibid.) also stresses how textbooks and teachers can serve to guide such superficial imitative reasoning. In contrast, creative mathematical reasoning involves novelty, plausibility and mathematical foundation, while creativity is seen as an orientation or disposition toward a mathematical activity that can be fostered broadly in school. This perspective is reflected in US Standards of the National Council of Teachers of Mathematics (NCTM, 2000) that recognize reasoning and proof as fundamental aspects of mathematics. 'People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove' (p. 56). Such reasoning can have many functions in mathematics, including verification, explanation, systematization, discovery, communication, construction of theory and exploration. This framework provides a specific focus on problem-solving, especially the phases formulated by Pólya (1954) and elaborated by Schoenfeld (1985). These are: reading the task (including noting conditions and goals), analysing (to understand, select perspective and perhaps reformulate), exploring (a broader and less structured search for information), planning, implementing (including evaluation of progress) and verifying.

The emphasis on questions of epistemology was set at the outset of the course by the first key question 'What is Mathematics?'. Consideration of this question was framed by the discussion concerning the distinction between *mathematical fallibilism* and *mathematical fundamentalism* from the start of the course of study. Recognition of the process of didactic transposition in which knowledge can mutate or be degraded led directly to the consideration of the quality of the knowledge at stake. In parallel, the analysis of the joint action in the classroom through the application of the JATD framework highlighted the significance of the *epistemic game* in particular. All of these aspects combined in such a way that the quality of the (mathematical) knowledge or *epistemic quality* emerged as a key issue—in other words, the quality of what the students came to know, to make sense of and be able to do.

Accordingly, the characteristics of mathematical fallibilism outlined above are associated with *high epistemic quality* whilst those characterizing mathematical fundamentalism are associated with *low epistemic quality*. It is further argued (Hudson et al., 2015) that high epistemic quality is promoted through an approach based on assessment *for* learning involving low stakes formative and self-assessment. This is engaging and motivating for individual learners and can create the conditions leading to a sense of enjoyment and fulfilment of mathematics as a creative human activity. In contrast, the excessive pressure from high stakes external testing and inspection and the associated heavy emphasis on memorization, drill and practice establish circumstances that can degrade epistemic quality into the mutated form of mathematical fundamentalism described earlier and lead to an experience for learners of mathematics that is fearful and anxiety-inducing, boring, demotivating and alienating from the subject itself.

Furthermore, in considering the relation between epistemic quality and powerful knowledge (Young, 2013) by framing it within a sociological theory of knowledge Hudson (2018) highlights the distinction between *knowing that* and *knowing how*. This draws on the work of Muller (2016) who distinguishes between two kinds of knowledge, i.e. propositional knowledge or *knowing that* and procedural knowledge or *knowing how* and it is argued that every area of the curriculum can be described in these terms. Furthermore, there are two different kinds of 'know how' knowledge that are important for the curriculum—inferential know how and procedural know how. Firstly, with regard to inferential know how, this is about 'knowing how the conceptual knowledge (the "know that") hangs together and how to negotiate the epistemic joints that link the various knowledge bits together' (Muller, 2016, p103). Secondly in relation to procedural know how, this points to



**Table 1.** Drivers and impacts of epistemic quality in mathematics.

High Epistemic Quality	Low Epistemic Quality
Involves an approach that presents mathematics as fallible, refutable and uncertain.	Involves an approach that presents mathematics as infallible, authoritarian, dogmatic, absolutist, irrefutable and certain.
Promotes problem solving, critical thinking, creative reasoning, the generation of multiple solutions and learning from errors and mistakes.	Emphasizes rule following and memorization of strict procedures, promotes algorithmic and memorized reasoning and right or wrong answers.
Promotes <i>knowing how</i> or procedural knowledge	Over emphasizes <i>knowing that</i> or propositional knowledge
Supported by diagnostic feedback through formative assessment for learning and self-evaluation of schools	Reinforced by excessive high stakes external testing, summative assessment and school league tables
Impact on students results engagement and motivation; enjoyment and fulfilment; mathematics seen as a creative human activity	Impact on students results in boredom and de-motivation; fear and anxiety and alienation from mathematics.

a more risky and uncertain kind of knowledge where the newcomer ‘learns how to find out new things, finds out which warrants and tests work under what circumstances, what the tolerances and limits are in real situations, forming new judgements that lead to solutions that work in the world’ (ibid, p103). This discussion is set within a national context in England in which there is a strong political steer towards a so-called ‘knowledge-rich’ or ‘knowledge-based’ curriculum (Wright, 2016). However, in this context knowledge is taken as given at the policy level and is strongly influenced by the thinking arising from the Core Knowledge Foundation, the resources of which have been imported from the USA and strongly promoted in English schools over recent years directly from the Department for Education in Westminster. As a result, there is a very strong emphasis on simply factual knowledge i.e. ‘knowing that’. Furthermore an evaluation of one of the key texts related to mathematics in Hudson (2018) illustrates an overemphasis on practice and memorization, a fragmented view of the subject and standard procedures reduced simply to rule following resulting in the content of low epistemic quality. These characteristics of high and low epistemic quality in school mathematics are summarized in Table 1:

## 5. Equitable access to quality education in school mathematics

Questions of equity and access in particular became central considerations of the JADE project as a whole (Hudson, Loquet, Meyer, & Wegner, 2018). This process was informed initially by the UNICEF/UNESCO (2013) report that followed the Global Thematic Consultation on Education in the Post-2015 Development Agenda (UNICEF/UNESCO, 2013).<sup>1</sup> This report focused on education with special reference to societal challenges and the need for worldwide responses from education systems to them. It pinpoints education as a ‘fundamental human right in itself as well as an enabling right, fostering the accomplishment of all other social, cultural, economic and political rights’. It also highlights the benefits of investment in quality education, among them the formation of responsible citizenship-oriented to principles of respect for life, human dignity and cultural diversity and thereby of more resilient and stable societies (UNICEF/UNESCO, 2013, p. 2). Thus, equitable quality education must be seen as a prior goal as defined in the UNESCO/UNICEF paper:

Universal access to quality education, training and learning is an essential prerequisite for individual empowerment, the development of equitable societies and the promotion of social justice. The provision of education contributes to the eradication of poverty, the promotion of social cohesion, good governance and participatory citizenship, and improved health and gender equality. The education consultation as well as the other thematic consultations noted that education is a catalyst for inclusive economic growth, equipping people with the training, skills and competencies needed to secure decent work and be productive citizen. Education also has a key role to play in conflict prevention and peacebuilding as well as in disaster preparedness and risk education. (UNICEF/UNESCO, 2013, p. 3)

This report calls for two main education-specific goals to be addressed as part of the future development framework: equitable access and equitable quality education (UNICEF/UNESCO,

2013, p. 36). Subsequently, these goals were reflected in the UN Sustainable Development Goal Number 4 (United Nations, 2015), which is to ensure inclusive and equitable quality education and promote lifelong learning opportunities for all. However, this is a challenge not only at the global level but also nationally for all contemporary societies and education systems. Of particular interest is the need for a better understanding of the conditions in which everyday teaching practices and classroom interaction produce equitable access to high-quality education, i.e. equal preconditions and chances of autonomous participation and success. In so doing, it is recognized that there is a need to consider issues of equity and access at the societal, institutional and classroom levels.

With regard to the issue of access to quality education in an international development context, this has generally been considered to be simply a question of school enrolment. However, as argued by Humphreys, Moses, Kaibo, and Dunne (2015) enrolment figures are an inadequate indicator of access because being enrolled in school does not necessarily mean being in school, and being in school does not necessarily mean being engaged in productive learning. For this reason, they identify four distinct stages of access:

- (i) access as enrolment;
- (ii) access as sustained attendance (sustained access);
- (iii) access to the classroom, once in school; and finally
- (iv) access to the curriculum, with pupils engaged in meaningful learning.

These collectively form access to 'good quality education' (ibid) as the ultimate goal which aligns with the concept of (high) epistemic quality. This argument is resonant with that of Morrow (2008) who offers the notion of *epistemic access* which has subsequently been used as a curriculum principle that maximizes the chances for all pupils to experience high-quality education in school mathematics. For example, in building on this notion in relation to primary education, du Plooy and Zilindile (2014, p. 198) relate this to the idea of 'meaningful or expanded' access. Accordingly, the concept of epistemic access is a key consideration in this study alongside that of epistemic quality.

## 6. Research design

### 6.1. The DMTPC project

The DMTPC project as a whole was set up within a didactical design research framework as described in Hudson et al. (2015) which aims to address the ways in which we might systematically create, test, evaluate and disseminate teaching and learning interventions that will have a maximum impact on practice and contribute significantly to the development of theory about teaching and learning. Empirical data in the main study were drawn from pre- and post-trial surveys of the teachers' perceptions, interviews with participants, observations of engagement in the online environment, document analysis of audits of classroom practice and action plans for and action research project reports from classroom inquiry. The interviews focussed on teachers' perceptions concerning their levels of confidence and competence in relation to teaching mathematics; their attitudes and beliefs in relation to mathematics as a subject and their expectations of the impact on pupil learning arising from this course of study.

### 6.2. The case study

The case study presented in this paper is based on analysis of classroom interaction in a series of mathematics lessons. It focusses on an action research project on the development of a topic-based approach to teaching and learning mathematics on the theme of 'The Rainforest' conducted by one teacher. Anna (a pseudonym) was attending the Masters level course developed as part of the DMTPC project and her project was assessed as being excellent. The text that follows in sub-section 6.4 and

section 7 is reproduced with agreement of the journal *Education & Didactique*, in which it was first published (Hudson, 2015).

### 6.3. Research questions

The main focus of this study is to explore the necessary conditions for ensuring equitable access to quality education in school mathematics and to consider what kinds of teaching and learning activities can be equitable for all students. The emphasis placed on quality education brings a focus on the didactic system and within this frame this study focuses on the following two aspects:

- (i) the quality of teacher-student(s) joint action and
- (ii) the epistemic quality of content

### 6.4. Anna's action research project

The action research project was framed within an overall approach to mathematical thinking based on one of the core texts of the course by Mason et al. (2010). Three broad phases are identified in responding to a mathematical question, are described as 'entry, attack, and review', as illustrated in Figure 1. These phases correspond to differing emotional responses to the question and also reflect whether or not progress is being made. Becoming aware of these phases is seen to enable the learner to deal more effectively with the situation.

Anna's action research project was based on addressing the question regarding the extent to which a topic-based approach to mathematics allows children to demonstrate their mathematical thinking. She was also interested in the extent to which topic-based mathematical questions allow children to verbalize their thinking and also the effect of such an approach on children's levels of engagement. This study took place in a five-class, rural primary school in Angus in North East Scotland, with a mixed-age class of Year 5 and Year 6 pupils, aged between 9 and 11 during the Spring and Summer terms in 2012. The chosen topic for Anna's action research project was 'The Rainforest' and, as a result, 23 pupils were given 4 questions to explore, analyse and record during a period of 3 weeks. The mathematical content was 'measurement', which primarily related to measuring length and weight. The four questions the pupils worked on were:

- How could we measure these life-sized insects accurately?
- How could we mark out the different layers of the rainforest in our playground?
- Can you compare the length of the River Tay and the Amazon River?
- Is there a relationship between the weight of an animal and the layer it lives on in the rainforest?

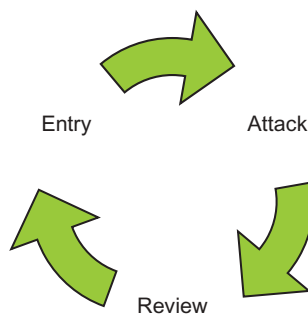


Figure 1. Entry, attack and review phases in responding to a mathematical question.

Accounts of happenings in teaching and learning situations were recorded by collecting data in three ways. First, children's talk was recorded informally during conversations with peers. Second, notes were made of what children said during class feedback sessions and comments on the lessons in feedback sessions were recorded. Further, observations were made of the levels of engagement within the class and various parts of the activities were filmed to watch and analyse at a later date.

### **6.5. Addressing the quality of teacher-student(s) joint action**

Each of the four questions formed the focal point for one of four lessons during the three-week period with each lesson running over several days. In this paper, extracts from Lesson 1 and Lesson 3 only are used for the analysis due to the extensive nature of the data relating to all four. Anna's account is then analysed using the theoretical framework offered by JATD. The quality of teacher-student(s) joint action is analysed firstly using the key concepts drawn from the JATD of 'game', 'contract' and 'milieu'.

## **7. Data analysis and interpretation**

### **7.1. Lesson 1: measuring insects**

Lesson 1 involved the question 'How could we measure these life-sized insects accurately?'. In her report, Anna states that the 'learning intention' was for the children to become confident in measuring with millimetres and to be able to convert between centimetres and millimetres. Thus, in terms of JATD, the epistemic game was to be able to measure confidently with millimetres and to be able to convert between centimetres and millimetres.

The pupils were shown two life-size pictures of an Atlas moth and a Brazilian Huntsman spider and could choose which insect they wanted to investigate. From the start of the lesson, children actively discussed in pairs which insect they would choose and why. During feedback, many responses indicating enthusiasm and interest in the learning game were given.

Gillian (Year 6 girl): *'I'm wanting to measure the spider to see if its legs are all the same size.'*

John (Year 5 boy, who is often disengaged.): *'I hate spiders. No way I'm measuring that! I'll do the butterfly'*

Jane (Year 5 girl.): *'Can I do the butterfly? Which bits should I measure though?'*

This comment and question led to the next part of the whole class discussion in which Anna posed the following question, i.e. which parts should we be measuring? This question is seen as part of the didactic game in terms of JATD.

Once again, the children engaged actively in the learning game and discussed this eagerly in pairs. It was noted at this point that all pupils were actively engaged. Anna saw this part of the lesson having clear correspondence to the 'Entry Phase' described by Mason et al. (2010) since the children were slowly determining what they knew already and were building up strategies to solve the task. The children came to the conclusion that they could measure the 'bits' of the insects they thought were important. In terms of the JATD, this phase may be seen as one in which the children were beginning to actively engage in the milieu, i.e. the system of material and symbolic objects in question that corresponds to the new knowledge the students are to acquire.

The didactic game and associated classroom interaction continued with Anna posing the next question:

Teacher: *'What will you use to measure?'*

At this point, a unanimous decision was made to use a ruler and one boy (Tony) was selected to demonstrate measuring one leg of the spider with a ruler. Tony selected a ruler, measured the leg from tip to tip and replied as follows:

Tony (Year 5 boy): *'It's about 8 centimeters.'*

The didactic game continued with Anna asking the next question and setting the next task:

Teacher: *'Does that tell us exactly how long this leg is? Discuss with your partner.'*

The children engaged in the learning game and 4 out of the 12 pairs realized it was not an accurate reading because the leg was bent in different places. One Primary 5 child then came up with the idea of moving the ruler in different directions and another with the idea of moving the paper. These responses were praised by the teacher but, after giving the children more thinking time, a Primary 6 girl (Josie), who usually had little self-confidence in her mathematical abilities, suggested using string to follow the leg and then to measure that. The teacher celebrated this idea and the other children agreed on this method. During the learning game, the pupils were actively listening to each other and keen to see the use of string to measure.

Anna continued by asking a further question and setting a further task as part of the didactic game:

Teacher: *'As a rainforest researcher, would "about 8 centimeters" be enough information? Discuss this in your pairs.'*

Whisperings of 'millimetres' could be heard in some pairs, particularly those with Primary 6 pupils present. The Primary 5 pupils had no formal experience with millimetres, so this was a completely new concept for them. After the feedback session, it was agreed that measurements must be taken in millimetres to be more accurate. Each pupil was then given a ruler and millimetres were discussed. As part of the actual epistemic game, the root of the word 'milli' was discussed and pupils were able to suggest related words such as millennium and millilitre. The children were then asked to measure their fingers and fingernails, etc. in millimetres and conversions were made between centimetres and millimetres. The teacher noted how all children were actively engaged throughout this practical measuring exercise, turning the rulers around to compare centimetres and millimetres, which can be seen in terms of their active engagement in the milieu.

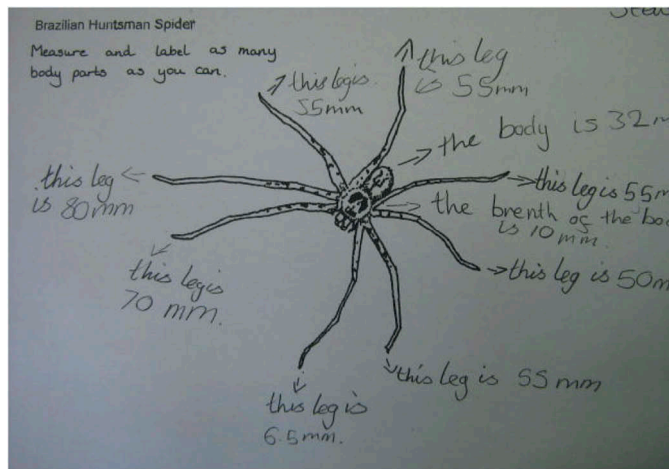
The pupils were then shown different ways of recording their findings by the teacher such as 1.1 cm, 1 cm, 1 mm and 11 mm as part of the ongoing didactic game. Children were then given a piece of string each and set the task to measure different parts of their chosen insect and record their findings in their own way. During the measuring process, the teacher also asked the children to verbalize their thinking and demonstrate their measuring by way of advancing the learning game. Most children chose to write their measurements entirely in millimetres.

In this first example illustrated in [Figure 2](#), a Year 6 girl (Kaye) wrote short sentences to describe her measurements and when asked by the teacher why she had used millimetres, she said, *'It's easier to see the numbers this way. It's weird, they're all (the spider's legs) different lengths mostly.'*

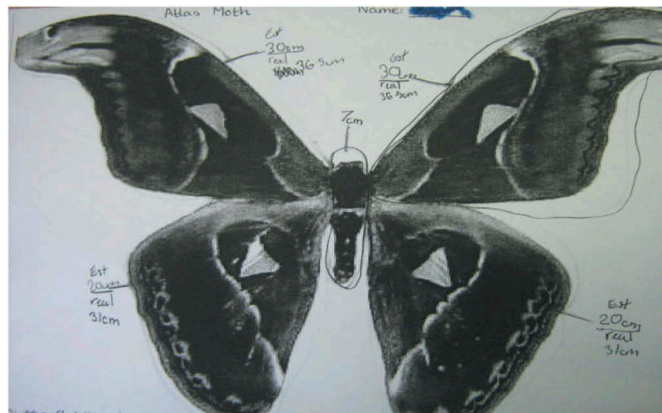
This demonstrates how Kaye was actively involved in the learning game and was concerned not only with measuring but also with clearly presenting her results as part of the actual epistemic game.

In the second example shown in [Figure 3](#), a Year 5 girl (Jenny) chose to estimate her answers first without being prompted. When asked about it, she replied, *'I wanted to try to use my eyes first to see how big the butterfly's wings were. I thought the top ones were bigger and I was right but my guesses were too small.'*

This demonstrates how Jenny was actively involved in the learning game and was concerned with both the estimation and measuring as part of the actual epistemic game.



**Figure 2.** Kaye's measurements of the spider.



**Figure 3.** Jenny's measurements of the moth.

The third example illustrated in Figure 4, was by a Year 5 boy (Paul), described by the teacher as a high achieving pupil, and who eagerly wanted to show the teacher his work. *'Mrs. A, I've done all three ways. They are totally the same on each side, look. Have I missed any bits?'*

This demonstrates how Paul was actively involved in the learning game and how he was able to use both centimetres and millimetres as part of the actual epistemic game.

The final example shown in [Figure 5](#), is by a Year 5 boy (John) who was described by the teacher as a low achiever in mathematics. He was slowly measuring the body of the butterfly and, on observation, was being meticulous. He recorded his findings in centimetres and in millimetres, saying that, *'The body is exactly six centimeters; I don't need millimeters for that one.'*

This demonstrates how John was actively involved in the learning game and was able to see the relationship between centimetres and millimetres, dispensing with the need to use the latter on this occasion, as part of the actual epistemic game.

Overall, the children were able to respond to the questions and used the parts of the insects in their verbal responses during the plenary. Many pupils were also able to participate in the learning game by commenting on the symmetrical qualities of the animals, which led into an extension of the didactic game through a discussion on symmetry. This aspect is discussed further in relation to







*"The Amazon is like a snake, so some people might be measuring it from when it's just a wee stream and some might just be measuring it from the start of the big river."*

This example can be seen as one in which Andrew was beginning to actively engage in the milieu.

As part of the didactic game, the children were then asked by Anna to present their findings in a sentence, paragraph, or diagram. Anna saw this as a successful exercise for observing and recording the children's understanding of kilometres in a relaxed but focused atmosphere. Some of the children's findings are listed below to illustrate the learning game:

A Year 5 boy (Joseph) simply noted that:

*"The Tay is shorter than the Amazon."*

Another Year 5 boy (Andrew) went further:

*"The Amazon is the second biggest river in the world at around an amazing 6400 kilometers long. So the Amazon is longer."*

A Year 6 boy (Keith) described a more extensive use of the Internet:

*"We only found our results in miles but we found an online converter and put them both into kilometers."*

A Year 5 girl (Karen) stated that:

*"We took away 193 from 6360 to find the difference between the two rivers."*

This comment intrigued the rest of the class and prompted the activity of repeating the calculation to see if Karen had calculated correctly as part of the actual epistemic game.

Anna describes how, following this activity, a Year 5 boy (John) wanted to explain the diagram he had created with his partner.

He stated, *'I drew the Amazon and the River Tay on my piece of paper. I measured the paper and it was about 300 millimeters so we narrowed it down and got that every 5 centimeters was about 1000 kilometers. The River Tay is only 186 kilometers so it's only that size.'*

She describes how he then pointed to the part of their diagram labelled 'River Tay' at the very bottom of the page shown in [Figure 6](#).

Anna noted how this comment and diagram not only confirmed John's 'understanding' of kilometres, but also how it demonstrated his 'understanding' of centimetres and millimetres too. Further, she noted how from the teacher's perspective it had been an excellent opportunity to not only see the children's mathematical thinking unfold in their diagram, but to also listen to their explanation and observe their enthusiasm, thus in terms of JATD illustrating her thinking behind the didactic game. The other children in the class were very interested in this and one Year 6 girl (Tracy) continued the learning game by commenting that:

*"The Tay is tiny compared to that, you could fit like, a hundred of the Tay into the Amazon!" (Tracy)*

Anna notes how this comment was explored and extended, which may be seen as part of the learning game, leading to her question, which further extended the didactic game:

*"How many times would the Tay fit into the Amazon River?" (Anna)*

In her subsequent presentation to the School Research conference some weeks after completing the course, Anna described this as a 'light bulb moment' and this aspect is discussed further in the concluding discussion to this paper.

As part of the ongoing learning game, one Year 6 boy (Michael) suggested that both the numbers should be rounded off to make it easier. In response, a Year 6 girl (Charlotte) said:

*'That would be... 190 kilometers?'*

Michael then replied: *'It'd be good to do it to 200, to the hundred instead.'*

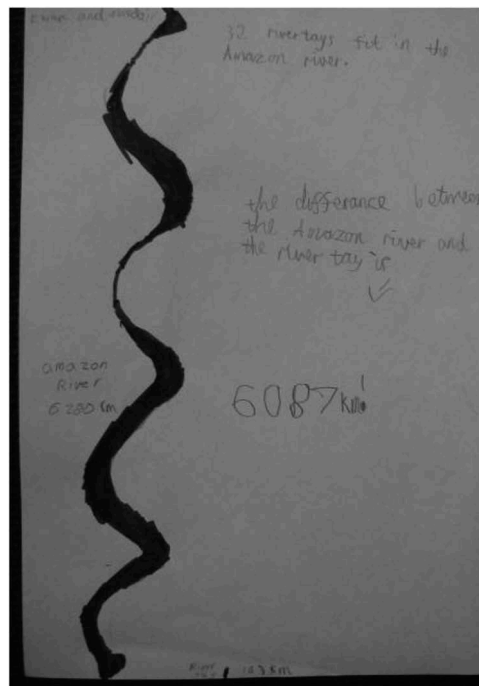


Figure 6. John's drawings of the River Tay and the River Amazon.

As part of the didactic game, Anna organized the class to vote on the two suggestions and it was decided to work to the nearest hundred, thus extending the epistemic game. Children were then set to task to solve this problem in pairs. Various methods were demonstrated, and two examples of the children's working are shown below. To finish the lesson, the results were discussed and strategies explained.

Here in Figure 7 and Figure 8 are two examples of the feedback as part of the learning game, which highlight the development of the children's mathematical thinking as part of the actual epistemic game.

*"I did 200,400,600,800 and 1000, so there's 5 River Tays in 1000. That means there's 30 in 6000 because 5 times 6 is 30, Then I just needed two 200s to get the 400 kilometers left. So the Amazon is about 32 River Tays."*

*"I did 6400 divided by 200 as a sum to find out how many River Tays I could get in the Amazon. I said, 'How many two hundreds are in 6, none, in 64 none, in 640, 3 with 40 left over. Then I put that 40 with the zero in the units column and got two hundreds in 400. That made 32.'"*

Anna describes how the discursive element of this lesson proved to be a very effective tool to assess the pupils' understanding and mathematical thinking and this aspect is discussed further in the final section.

## 8. Discussion

In both of these lessons, we are able to share Anna's observations of how the children actively engaged in the milieu, i.e. the system of material and symbolic objects in question that corresponds to the new knowledge the students are to acquire. Both lessons also demonstrate the way in which the didactic contract provides a common background between the teacher and the

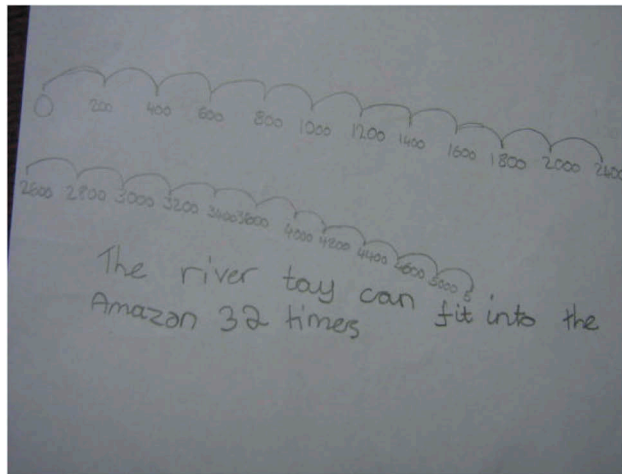


Figure 7. First example of a child's working.

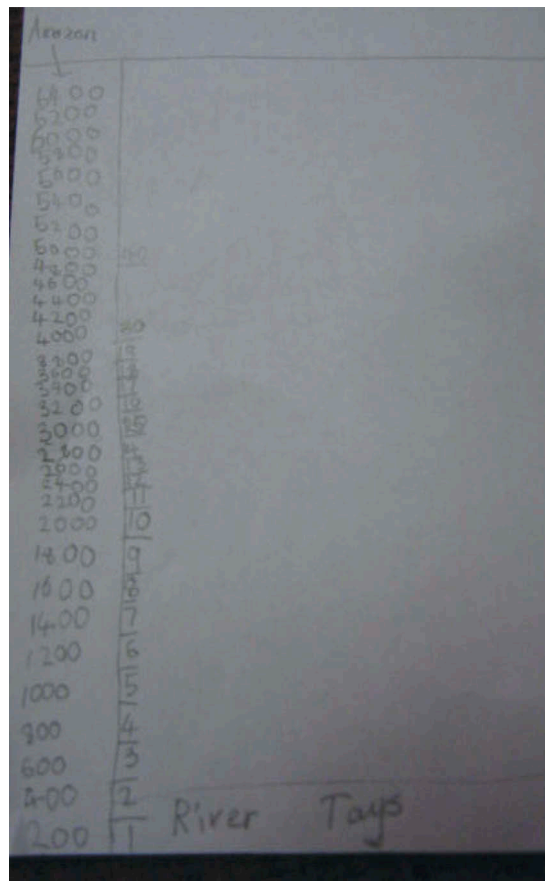


Figure 8. Second example of a child's working.

students against which the didactic transactions occur and also the dialectic between the contract and milieu.

Anna's analysis represents the actual epistemic game based as it is an analysis of class practices as they occur in-situ. Her analysis of the didactic game relative to each lesson provides a detailed description of the milieu. For example, in Lesson 1 she provides a picture of the manner in which children engage actively in the milieu from the point at which they came to the conclusion that, through a collective process of creative reasoning, they could choose to measure the 'bits' of the insects they thought were important through to the point at which they were able to respond to the questions and use the parts of the insects in their verbal responses during the plenary. Moreover, she highlights how this engagement in the milieu corresponded to the 'Entry Phase' (Mason et al., 2010) of mathematical problem-solving by reflecting on previous knowledge and by building up creative problem-solving strategies to solve the task. She also notes that many pupils were also able to comment, as part of the learning game, on the symmetrical qualities of the insects, which led into a discussion on symmetry, thus highlighting the potential to extend the source epistemic game through the use of the open-ended topic-based approach combined with effective teacher questioning.

In relation to the epistemic quality of the content, Anna notes that in Lesson 3, as with the previous lessons, it was evident the children were able to take their investigation further according to their level of understanding and previous knowledge. A particularly significant aspect of Lesson 3 is the incident Anna described as a 'light bulb moment' in her subsequent conference presentation. This marked a moment in time when the source epistemic game became modified and extended due to the creative reasoning that was enabled through the open-ended nature of the actual epistemic game combined with Anna's didactic ability to respond in the moment and build on her pupils' remarks. Anna notes how Tracy's creative reasoning about how tiny The River Tay is compared to the Amazon and that '... you could fit like, a hundred of the Tay into the Amazon!' was explored and extended leading to the question 'How many times would the Tay fit into the Amazon River?' Anna's question as part of the didactic game extended the actual epistemic game in a very spontaneous and creative way. As a result of extending the epistemic game in this way the epistemic quality of the content was greatly enhanced into one which was concerned with estimation, scale and calculations involving addition, subtraction, division and multiplication with up to four-digit numbers. Of course, the initial epistemic game contained the potential for this extension given that it was based on the children developing the ability to 'demonstrate an understanding of the enormity of the Amazon River in relation to the River Tay'.

In relation to the quality of teacher-student(s) joint action, Anna describes through her analysis how the discursive element of this lesson proved to be a very effective tool to assess the pupils' understanding and mathematical thinking. In terms of the JATD, the lesson as a whole could be seen as one in which the children actively engaged in the milieu, the way in which the didactic contract provides a common background, and also the way in which dialectic operated between the two through effective teacher questioning and the resulting responses from the students. Anna's original 'learning intention' for this didactic game was for the children to become confident in measuring with millimetres and be able to convert between centimetres and millimetres. However, the children's mathematical thinking was extended through a process of creative reasoning to consideration of the concept of scale, which involved estimation and also calculations entailing addition, subtraction, division and multiplication with up to four-digit numbers. Since all pupils recognized the vast difference between the two rivers, their natural inquisitiveness and natural orientation towards creative reasoning drove them to extend their own thinking and the thinking of others. The question in Lesson 3 in particular developed tremendously throughout the lesson. It was evident in this study that the children had very differing prior knowledge and experiences to bring to the problem-solving elements of the tasks. Moreover, the mathematics became more accessible to all due to their ability to visualize the problems and to engage in collective creative reasoning. This resulted from the high quality of the teacher-student joint action

and the high epistemic quality of the content, which in turn produced an evolution in mathematical thinking and high-quality epistemic access *for all*.

In conclusion, the study that is the focus of this paper has addressed some of the challenges of UN SDG 4 to ensure inclusive and equitable quality education for all in relation to school mathematics. With regard to ensuring equitable access to quality education, it is argued that this should not be considered to be simply a question of school enrolment. As argued by Humphreys et al. (2015) it is the case that enrolment figures are an inadequate indicator of access because being enrolled in school does not necessarily mean being in school, and being in school does not necessarily mean being engaged in productive learning. Rather, it is necessary to have access to the curriculum, with pupils engaged in meaningful learning. In the terms used by Morrow (2008), it is necessary that the curriculum be underpinned by the curriculum principle of epistemic access that maximizes the chances for all pupils to experience high-quality education in school mathematics. Accordingly, in relation to the goal of achieving high-quality education, it is necessary to consider the epistemic quality of the content in terms of what students come to know, make sense of and be able to do in school mathematics. In particular it is necessary to achieve high epistemic quality and this can be characterized by an approach that presents mathematics as fallible, refutable and uncertain; that promotes problem-solving, critical thinking, creative reasoning, the generation of multiple solutions and learning from errors and mistakes; that promotes *knowing how* or procedural knowledge and which is supported by diagnostic feedback through formative assessment *for* learning and self-evaluation of schools. The impact on students from such an approach is most likely to result in engagement and motivation; enjoyment and fulfilment and mathematics being seen and experienced as a creative human activity. As highlighted earlier, the necessary analytic study is only part of the researcher's work (Sensevy, 2011) and the main purpose of JATD is to grasp the dynamics and the unity of the joint action. From within an epistemology of 'exemplar' as defined by Kuhn (1962), the role of Anna's action research project is presented as potentially constituting an 'exemplary' case of learning from the situation involving high-quality teacher-student(s) joint action and high epistemic quality of content in school mathematics. This is used to illustrate how mathematics can become more accessible and inclusive thus leading to an evolution in mathematical thinking and high-quality epistemic access *for all*.

## Note

1. The Global Thematic Consultation on Education in the Post-2015 Development Agenda was led by UNESCO and UNICEF. Support is given by the governments of Senegal, Canada, and Germany as well as by the William and Flora Hewlett Foundation. The objective of this consultation was to assess the progress, but also the remaining challenges in meeting the education-related Millennium Development Goals adopted in 2000 and the broader Education for All goals (UNICEF/UNESCO, 2013, p. 2).

## Disclosure statement

No potential conflict of interest was reported by the author.

## Funding

This work was supported by the Scottish Government [Developing Mathematical Thinking in the Primary Classroom].

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